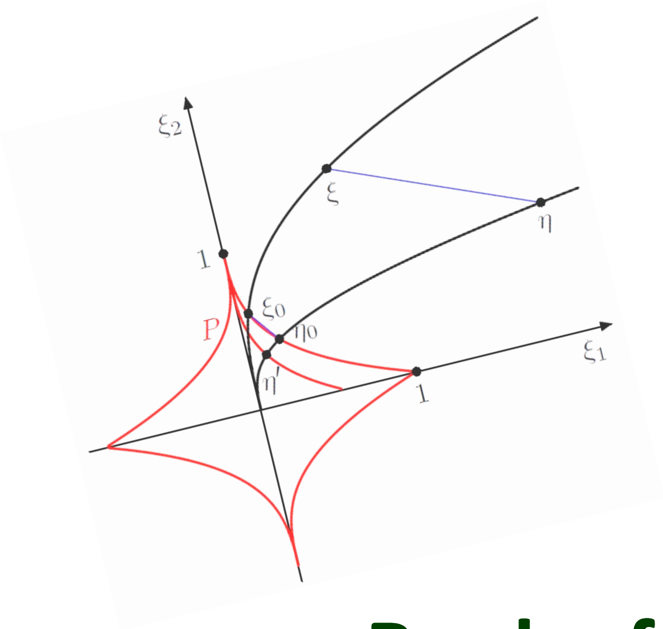
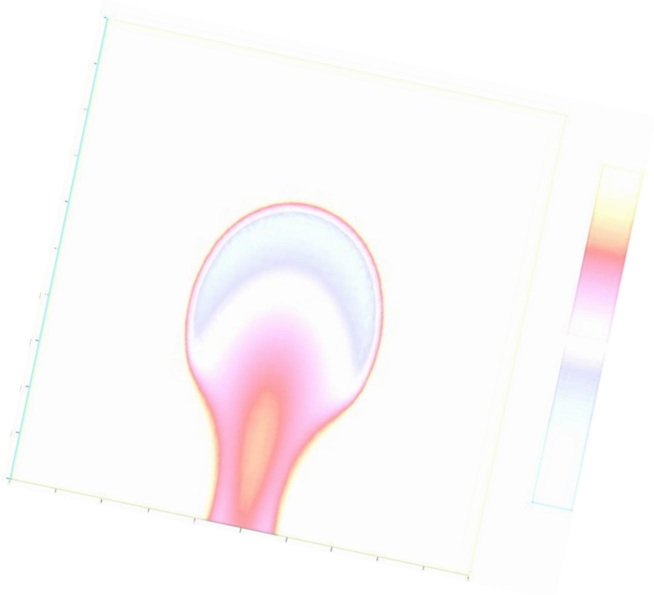
WeConMApp

Weak convergence methods and applications

International Workshop on PDEs: analysis and modelling





Book of abstracts

Zagreb, June 17-20, 2018

Scientific and organising committee

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Department of Mathematics, Faculty of Science, University of Zagreb

Venue

Department of Mathematics, Faculty of Science, University of Zagreb
Bijenička cesta 30, Zagreb
Room **A102** (first floor; east wing of the building)

International workshop on PDEs: Analysis and modelling -- Schedule --

17.06.
17:00 – 18:00 registration

18.06.	
8:45-9:00	registration and opening
9:00-9:50	Muha
10:00-10:30	coffee break
10:30-10:55	Ljulj
11:00-11:25	Scandone
11:30-11:55	Kunštek
12:00-13:30	lunch break
13:30-14:20	Dapogny
14:30-14:55	Raguž
15:00-15:30	coffee break
15:30-16:20	Burazin
16:30-16:55	Crnjac

19.06.	
9:00-9:50	Wollner
10:00-10:30	coffee break
10:30-10:55	Nakić
11:00-11:25	Gallone
11:30-11:55	Vojnović
12:00-13:30	lunch break
13:30-14:20	Bukal
14:30-14:55	Slijepčević
15:00-15:30	coffee break
15:30-16:20	Šohinger
16:30-16:55	Olgiati

20.06.	
9:00-9:50	Seleši
10:00-10:30	coffee break
10:30-10:55	Rabar
11:00-11:25	Bojanjac

All lectures will be held in room **A102**,
which is located on the 1st floor of the Dept. of Mathematics (Bijenička c. 30).

HIGHER-ORDER DIFFUSION EQUATIONS AND RELATED FUNCTIONAL INEQUALITIES

MARIO BUKAL

Unlike second-order diffusion equations, which are physically and probabilistically more grounded, higher-order diffusion equations typically arise as approximation of physical models. For example, fourth-order thin film equations arise in lubrication approximation of thin viscous fluids describing the dynamics of the relative fluid height. In particular, we will consider fourth- and sixth-order equations approximating quantum effects in nano-semiconductors. On the other hand, functional inequalities like logarithmic Sobolev or Nash inequality are very important tools in the analysis of PDE's and in probability theory. Surprisingly, there are deep connections between second-order diffusion equations and functional inequalities in sharp form. Probably the most famous and most studied connection is the Bakry-Émery method. In this talk we will present Toscani's approach [1, 2], which is based on an information-theoretic property called concavity of the entropy power [3]. Showing the concavity of the entropy power along solutions to the fourth-order quantum diffusion equation, we provide higher-order logarithmic Sobolev and Nash inequalities in a sharp form.

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HOMOGENIZATION OF KIRCHOFF-LOVE PLATE EQUATION AND COMPOSITE PLATES

KREŠIMIR BURAZIN [1], JELENA JANKOV [2], AND MARKO VRDOLJAK [3]

General, non-periodic homogenization theory is well developed for second order elliptic partial differential equations, where the key role plays the notion of H-convergence. It was introduced by Spagnolo through the concept of G-convergence (1968) for the symmetric case, and further generalized by Tartar (1975) and Murat and Tartar (1978) for non-symmetric coefficients under the name H-convergence. Some aspects for higher order elliptic problems were also considered by Zhikov, Kozlov, Oleinik and Ngoan (1979). Homogenization theory is probably the most successful approach for dealing with optimal design problems (in conductivity or linearized elasticity), that consists in arranging given materials such that obtained body satisfies some optimality criteria. Mathematically, this is usually expressed as minimization of some (integral) functional under some (PDE) constrains.

Motivated by a possible application of the homogenization theory in optimal design problems for elastic plates, we adapt the general homogenization theory for Kirchoff-Love elastic plate equation, which is a fourth order elliptic equation. In addition to the compactness result, we prove a number of properties of H-convergence, such as locality, irrelevance of the boundary conditions, corrector results, etc. We also calculate the first correction in the small-amplitude homogenization limit of a sequence of periodic tensors describing material properties of the plate, discuss smooth dependence of H-limit on a parameter, and calculate H-limit of a periodic sequence of tensors.

Using this newly developed theory, we derive expressions for elastic coefficients of composite plate obtained by mixing two materials in thin layers (known as laminated materials). Moreover, we also derive optimal bounds on the effective energy of a composite material, known as Hashin-Shtrikman bounds.

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SHAPE OPTIMIZATION OF A LAYER BY LAYER MECHANICAL CONSTRAINT FOR ADDITIVE MANUFACTURING

GRÉGOIRE ALLAIRE [1], CHARLES DAPOGNY [2], RAFAEL ESTEVEZ [3], ALEXIS FAURE[3], AND GEORGIOS MICHAILIDIS [3]

In this presentation, we introduce a new constraint functional for shape optimization problems, which enforces the constructibility by means of additive manufacturing processes, and helps in preventing the appearance of *overhangs* - large regions hanging over void which are notoriously difficult to assemble using such technologies.

The proposed constraint relies on a simplified model for the construction process: it involves a continuum of shapes, namely the *intermediate shapes* corresponding to the stages of the construction process where the total, *final* shape is erected only up to a certain level.

The shape differentiability of this constraint functional is analyzed - which is not a standard issue because of its peculiar structure. Several numerical strategies and examples are then presented.

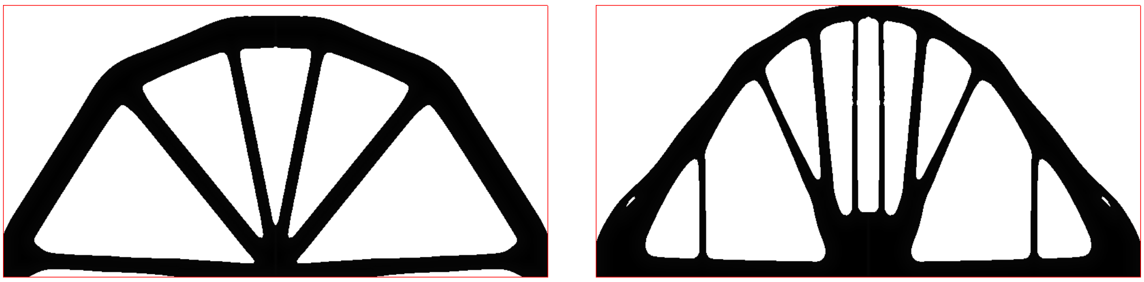

FIGURE 1. *(Left) Optimized 2d bridge with respect to the compliance, under a volume constraint; (right) optimized 2d bridge, with an additional constraint on the presence of overhangs.*

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WEAK-STRONG UNIQUENESS FOR A FLUID-RIGID BODY INTERACTION PROBLEM

BORIS MUHA

We consider a coupled PDE-ODE system describing the motion of the rigid body in a container filled with the incompressible, viscous fluid. The fluid and the rigid body are coupled via no-slip or Navier's slip boundary condition. We prove that the local in time strong solution is unique in the larger class of weak solutions on the interval of its existence. To the best of our knowledge this is the first weak-strong uniqueness result in the area of fluid-structure interaction. This is joined work with N. V. Chemetov, Š. Nečasovà and A. Radošević.

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CHAOS EXPANSION SOLUTIONS OF SINGULAR STOCHASTIC DIFFERENTIAL EQUATIONS

DORA SELEŠI

The probabilistic modeling of uncertainty plays an important role in a variety of scientific fields. In particular, stochastic partial differential equations have been deeply studied in recent years as they arise in physical and biological sciences, engineering, optimal control, mathematical finance and insurance, etc. If the initial data, the driving terms or the coefficients of a physical model are unknown, uncertain or vary in a random way, the uncertainty can be quantified by probabilities and stochastic methods can be used to develop an appropriate model. A typical example is white noise, which due to its constant spectral density models irregular fluctuations, the lack of all information or control over the real perturbation in the model.

In this talk we consider several parabolic, hyperbolic and elliptic problems for suitable differential operators, acting by the Wick product. The Wick product is involved due to the fact that we allow random terms to be present both in the initial conditions, the driving forces, input data, as well as in the coefficients of the involved operators. Due to their singular/irregular behavior, solutions to these equations have to be sought in spaces of generalized stochastic processes. We implement the Wiener-Itô chaos expansion method combined with operator semigroup theory to prove existence and uniqueness of solutions for these classes of SPDEs. Using this method the original SPDE is transformed into a lower triangular infinite system of PDEs that can be solved recursively. Summing up the obtained coefficients and proving the convergence of the obtained series one arrives to the solution of the initial SPDE.

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GIBBS MEASURES OF NONLINEAR SCHRÖDINGER EQUATIONS AS LIMITS OF MANY-BODY QUANTUM STATES IN DIMENSION $d \leqslant 3$

JÜRG FRÖHLICH [1], ANTTI KNOWLES [2], BENJAMIN SCHLEIN [3], AND VEDRAN ŠOHINGER [4]

Gibbs measures of nonlinear Schrödinger equations are a fundamental object used to study low-regularity solutions with random initial data. In the dispersive PDE community, this point of view was pioneered by Bourgain in the 1990s. We prove that Gibbs measures of nonlinear Schrödinger equations arise as high-temperature limits of appropriately modified thermal states in many-body quantum mechanics. We consider bounded defocusing interaction potentials and work either on the d-dimensional torus or on $\mathbb{R}^d$ with a confining potential. The analogous problem for $d = 1$ and in higher dimensions with smooth non translation-invariant interactions was previously studied by Lewin, Nam, and Rougerie by means of entropy methods. In our work, we apply a perturbative expansion of the interaction, motivated by ideas from field theory. The terms of the expansion are analyzed using a diagrammatic representation and their sum is controlled using Borel resummation techniques. When $d = 2, 3$, we apply a Wick ordering renormalization procedure. Moreover, in the one-dimensional setting our methods allow us to obtain a microscopic derivation of time-dependent correlation functions for the cubic nonlinear Schrödinger equation.

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OPTIMAL CONTROL OF PHASE-FIELD FRACTURE EVOLUTION

WINNIFRIED WOLLNER

Within this talk, we will address optimization problems governed by phase-field fracture/damage processes. The presence of an irreversibility of the fracture growth gives rise to a nonsmooth system of equations. To derive optimality conditions we introduce an additional regularization and show that the resulting optimization problem is well-posed [1], utilizing Stampaccia's cutoff argument [2]. We will discuss first order necessary optimality conditions and the corresponding constraint qualifications needed. Finally, we will see, that a quadratic approximation to the optimization problem is always solvable.

Further, to allow for the analysis of finite dimensional approximations to this problem, we provide a new regularity resulting providing improved differentiability of elliptic systems with non-smooth coefficients [3]. The result is based on Šneĭberg's stability theorem [4] and improves on past regularity results by [5] for improved differentiability of scalar problems and [6] on improved integrability in systems.

We will finally give an outlook on the resulting discretization error estimates [7], and discuss the limit in the regularized irreversibility condition [8].

This is joint work with R. Haller-Dintelmann, H. Meinlschmidt, M. Mohammadi, I. Neitzel and T. Wick.

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HOMOGENIZATION OF ELECTROMAGNETIC SCATTERING PROBLEM IN PERIODIC MATERIAL

DARIO BOJANJAC

In this work, homogenization of Maxwell's equations inside periodic heterogeneous microstructure in frequency domain is considered. In electrical engineering, especially in radioengineering, modelling and analysis of electromagnetic problems is usually made in frequency domain. Engineers are more interested in signal spectrum, or how some portions of spectrum behaves in their system than in time domain simulations. Reason for that lies in material behavior which is usually a function of frequency [1]. Using periodic structures as building blocks of lenses, metasurfaces, filters and other objects is very popular in the last 20 years [2]. In order to model and produce artificiall objects it is necessary to understand homogenization limit of Maxwell's equations and behaviour of effective parameters according to frequency of incoming wave.

In this paper we will focus on electromagnetic scattering problem on heterogeneous object, lens, made of periodic inclusions. Using Two-Scale Convergence [3] we will obtain unit cell problems that we treat numerically. All simulations shown in this talk are made in FEniCS [4].

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OPTIMALITY CRITERIA METHOD FOR OPTIMAL DESIGN PROBLEMS

KREŠIMIR BURAZIN [1], IVANA CRNJAC [2], AND MARKO VRDOLJAK [3]

In optimal design problems the goal is, for a given domain, two (or more) materials and its proportions, to find rearrangement of these materials within the body that optimises its properties with respect to some optimality criteria. The performance of the mixture is usually measured by an integral functional, while optimality of the mixture is achieved through minimization or maximization of this functional, under constraints on amount of materials and PDE constraints that underlay involved physics.

We consider multiple-state optimal design problems from conductivity point of view, where thermal (or electrical) conductivity is modeled with stationary diffusion equation and restrict ourselves to domains filled with two isotropic materials. Since the classical solution usually does not exist, we use relaxation by the homogenization method [2] in order to get a proper relaxation of the original problem.

One of numerical methods used for solving these problems is the optimality criteria method, an iterative method based on optimality conditions of the relaxed formulation. In the case of a single-state problem this method is described in [1]. Regarding multiple-state problems, in [3] a variant of the optimality criteria method is introduced, based on the optimality conditions derived in [1]. It appears that this variant works properly for maximization of conic sum of energies, but fails for the minimization of the same functional.

We rewrite optimality conditions for relaxed problem and develop a variant of optimality criteria method suitable for energy minimization problems. We also prove convergence of this method in a special case when a number of states is less then the space dimension and in the spherically symmetric case. Presented method can be adapted to similar problems in the context of linearized elasticity.

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DIRAC OPERATORS WITH COULOMB INTERACTION

MATTEO GALLONE

In relativistic quantum mechanics it is of central role the study of the Dirac equation. In describing the dynamics of a 1/2-spin Fermion in a Coulombian electrostatic field, for a sufficently large coupling constant, one faces the problem of the non-trivial self-adjoint realisations of the formal operator. In this talk I will present the state-of-art of the problem including recent results obtained in collaboration with Alessandro Michelangeli and some ongoing project in collaboration also with Marko Erceg.

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SHAPE DERIVATIVE METHOD AND APPLICATION TO OPTIMAL DESIGN PROBLEMS

PETAR KUNŠTEK[1] AND MARKO VRDOLJAK [2]

A multiple state optimal design problem for stationary diffusion equations with two isotropic phases is considered. Better conductivity is represented with β and worse with α. Distribution of phase α is denoted by characteristic function χ, so overall conductivity can be written by $A = \chi\alpha I + (1-\chi)\beta I$ and state equations are uniquely determined by temperatures $u_1, ..., u_m$:

$$\begin{cases} -\mathrm{div}(A\nabla u_i) = f_i \\ u_i \in H_0^1(\Omega) \end{cases} \qquad i = 1, ..., m.$$

Here, the right-hand sides $f_1, ..., f_m \in \mathrm{H}^{-1}(\Omega)$ are given, and the aim is to maximize a conic sum of energies obtained for each state problem. Commonly, optimal design problems do not have solutions (such solutions are usually called *classical*).

By analysing the optimality conditions we are able to show that in the case of annulus, the solution is unique, classical and radial.

The presence of classical solutions give opportunity to test different numerical methods i.e. compare rates of convergence, stability or check for possible errors. For demonstration, a shape gradient method was implemented in `Freefem++`. Based on shape derivative, method creates vector field which moves interface between phases in order to increase value of object function. We have observed stable convergence under tests created from optimal design problems with classical solutions.

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3D STRUCTURE – 2D PLATE INTERACTION MODEL

MATKO LJULJ [1] AND JOSIP TAMBAČA [2]

In this paper we rigorously derive models for interaction of a linearized three-dimensional elastic structure with a thin elastic layer of thickness ε and of possibly different material attached to it. Furthermore the attached thin material is assumed to have the elasticity coefficients which are of order $1/\varepsilon^p$, for $p \geq 0$ with respect to the coefficients of the three-dimensional body. In the limit five different models are obtained with respect to different choices of p. Moreover a three-dimensional–two-dimensional model is proposed which has the same asymptotics as the original three-dimensional problem. This is convenient for applications since one do not have to decide in advance which limit model to use.
This is a joint work with J. Tambača.

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COST OF THE NULL-CONTROLLABILITY OF PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

IVICA NAKIĆ

We will present a strategy for obtaining explicit estimates on the cost of the null-controllability for parabolic PDEs by using the so-called spectral inequalities. This will be applied to obtain estimates on the cost of null-controllability for the control heat equation in $\mathbb{R}^d$ with a heat generation term. We will also show how this cost behaves in certain homogenization and dehomogenization limits.

This talk is based on a joint work with Martin Tautenhahn, Matthias Täufer and Ivan Veselić.

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EFFECTIVE PROPERTIES OF MULTI-COMPONENT BOSONIC SYSTEMS

ALESSANDRO MICHELANGELI [1], PHAN THÀNH NAM [2], AND ALESSANDRO OLGIATI [3]

I will present results on the effective ground state and dynamical properties of a quantum system containing different bosonic species. After defining the many-body Hamiltonian in the Gross-Pitaevskii regime, I will show that the leading order of the ground state energy is captured by the minimum of a suitable effective functional. Moreover, the ground state exhibits condensation, in the sense of reduced marginals, on the minimizer of the same functional. Such results motivate and justify the assumptions for our theorems on the effec- tive dynamical description. I will show that the linear Schrödinger time evolution of multi-species systems preserves condensation. Moreover, the many-body dynamics is effectively well approximated by a system of non-linear Schr ödinger equations describing the evolution of the condensate wave-functions.

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THE TRANSPORT SPEED AND OPTIMAL WORK IN PULSATING FRENKEL-KONTOROVA MODELS

BRASLAV RABAR [1] AND SINIŠA SLIJEPČEVIĆ [2]

We consider a generalized one-dimensional chain in a periodic potential (the Frenkel-Kontorova model), with dissipative, pulsating (or ratchet) dynamics as a model of transport when the average force on the system is zero. We find lower bounds on the transport speed under mild assumptions on the asymmetry and steepness of the site potential. Physically relevant applications include explicit estimates of the pulse frequencies which maximize transport. The bounds explicitly depend on the pulse period and number-theoretical properties of the mean spacing. The main tool is the study of time evolution of spatially invariant measures in the space of measures equipped with the L^1-Wasserstein metric.

References

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SOME RESULTS ON REGULARITY OF FINITE-ENERGY SEQUENCES OF CAHN-HILLIARD FUNCTIONAL WITH NON-COERCIVE TWO-WELL POTENTIAL IN ONE DIMENSION

ANDRIJA RAGUŽ

In this presentation we extend Leoni's results in [5]. We obtain some regularity results for finite-energy sequences of Cahn-Hilliard functional in one dimension for certain non-coercive classes of two-well potentials W. We also present some open problems and formulate some conjectures.

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NON-LINEAR SCHRÖDINGER EQUATIONS WITH POINT INTERACTIONS

RAFFAELE SCANDONE

A central topic in mathematical physics is the rigorous investigation of many body quantum systems subject to very short range interactions. The dynamics of such systems can be efficiently described by non-linear Schrödinger equations with singular potentials. In this talk, I will discuss a recent result on the well-posedness of the Hartree equation with a point interaction in $\mathbb{R}^3$, in a suitable class of singular Sobolev spaces. I will also discuss various open problems.

This is joint work with G. Dell'Antonio, V. Georgiev, F. Iandoli, A. Michelangeli, A. Olgiati and K. Yajima.

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ERGODIC ATTRACTORS AND ALMOST-EVERYWHERE ASYMPTOTICS OF SCALAR SEMILINEAR PARABOLIC DIFFERENTIAL EQUATIONS

SINIŠA SLIJEPČEVIĆ

We consider dynamics of scalar semilinear parabolic equations on bounded intervals with periodic boundary conditions, and on the entire real line, with a general nonlinearity $g(t, x, u, u_x)$ either not depending on t, or periodic in t. While the topological and geometric structure of their attractors has been investigated in depth, we focus here on ergodic-theoretical properties. The main result is that the union of supports of all the invariant measures projects one-to-one to $\mathbb{R}^2$. We rely on a novel application of the zero-number techniques with respect to evolution of measures on the phase space, and on properties of the flux of zeroes, and the dissipation of zeroes. As an example of an application, we prove uniqueness of an invariant measure for a large family of considered equations which conserve a certain quantity ("mass"), thus generalizing the results by Sinai for the scalar viscous Burgers equation.

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DEFECT DISTRIBUTIONS RELATED TO CLASSES OF Λ-TYPE WEIGHTS

JELENA ALEKSIĆ [1], STEVAN PILIPOVIĆ [2], AND IVANA VOJNOVIĆ [3]

H-distributions (micro-local defect distributions) are introduced in [1] for weakly convergent sequences in the dual pair of $L^p - L^q$ spaces. In [2] we consider H-distributions in the case of unbounded multipliers (symbols) for sequences in the dual pair of Bessel potential spaces.

Following [2] we construct defect distributions which correspond to the space of symbols which are determined by the more general weight functions ([3]), for example of the form

$$\Lambda(\xi) := \sqrt{1 + \sum_{i=1}^{d} \xi_i^{2m_i}},\ \xi \in \mathbb{R}^d,$$

where $m = (m_1, \dots, m_d) \in \mathbb{N}^d$ and $\min_{1\le i\le d} m_i \ge 1$.

The results are applied in the analysis of a class of linear differential equations involving such weights as symbols. We analyze the existence of a solution for an equation with a sequence of weak solutions which corresponds to the sequence of approximating equations.

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