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Classical Optimal Design in Two-phase Conductivity Problems

Abstract

We consider multiple state optimal design problems for stationary diffusion in the case of two isotropic phases, aiming to maximize a conic combination of energies. It is well known that for problems with one state equation, there are relaxed solutions corresponding to simple laminates at each point of the domain. As a consequence, one can write down a simpler relaxation, written only in terms of the local proportion of given materials. For multiple state optimal design problems we prove an analogous result in the spherically symmetric case. This simpler relaxation problem is represented by a convex-concave minimax problem and its solution can be characterised by necessary and sufficient optimality conditions.

The optimality conditions are further analyzed on some examples. If the domain is a ball, the presented method enables explicit calculation of an optimal design which is, in most cases, unique and classical. However, an example with non-unique solution is presented as well.