Stability of Observations of Partial Differential Equations under Uncertain Perturbations

Martin Lazar
University of Dubrovnik

For a differential operator P_1 we analyse the problem of determining the initial energy for the equation $P_1 u = 0$ by observing an additive perturbation of a corresponding solution. It turns out that existing observability results for both the wave and the Schrödinger equation remind stable under robust, additive perturbations determined by a differential operator P_2, which can be chosen almost arbitrary, as well as corresponding initial data.

This generalises the results on averaged control recently obtained by the authors, in which the system under consideration consists of operators of the same type, and the initial data of all the components coincide as well.

The applied techniques employ tools of microlocal analysis, specially the localisation principle for microlocal defect measures or H-measures. However, the mentioned tool turn to be inappropriate when studying a system consisting of two parabolic operators, and a variant, better adapted to a study of parabolic problems is required. This one is found in recently introduced parabolic H-measures, capable of distinguishing different parabolic operators.

Relation of the results to the control theory will be discussed as well.