Exact solutions of multiple state optimal design problems

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We consider multiple state optimal design problems with elliptic state equations in the case of two isotropic phases, aiming to minimize a weighted sum of compliances. It is well-known that such problems do not have *classical* solutions, and thus a relaxation is needed by introducing generalized materials. We consider (proper) relaxation by the homogenization method which consists in introducing *composite materials*, which are mixtures of original materials on the micro-scale.

It is well known that for conductivity problems with one state equation, there exist relaxed solutions which correspond to simple laminates at each point of the domain. As a consequence, one can write down a simpler relaxation, ending by a convex minimization problem.

For multiple state optimal design problems this does not hold in general, but we derive analogous result if the number of states is strictly less than dimension of domain, as well as for arbitrary number od states in the spherically symmetric case. In both cases we prove that solving relaxed problem is equivalent to solving the simpler (relaxed) problem. Since this simpler relaxation is a convex optimization problem, one can easily derive the necessary and sufficient conditions of optimality and use them to calculate the optimal design. In the spherically symmetric case, we also prove that there exists a radial optimal design, and discuss its uniqueness in the case when domain is a ball. We demonstrate this procedure on some examples of optimal design problems, where the presented method enables us to explicitly calculate the unique solution.

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