Quantitative unique continuation and Wegner estimate for random Schrödinger operators

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In this talk we show how quantitative unique continuation principles can be applied to prove a Wegner estimate for the random Schrödinger operators, in particular to the random breather model. We also compare our result with earlier results on Wegner estimates for random Schrödinger operators.

Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space. The random breather model is in its simplest form given by the family of Schrödinger operators

$$H_\omega = -\Delta + \sum_{k \in \mathbb{Z}^d} \chi_{B_{r_j(\omega)}}(\bullet - k), \quad \omega \in (\Omega, \mathcal{A}, \mathbb{P}),$$

on $L^2(\mathbb{R}^d)$, where Δ denotes the discrete Laplacian and $\chi_{B_{r_j(\omega)}}$ the characteristic function of the ball with radius $r_j(\omega)$ and center zero. We assume that the random variables r_j are independent identically distributed with uniform distribution on $[0, \omega_+]$, $\omega_+ < 1/2$. Let $H_{\omega,L}$ denote the restriction of H_ω to the set $\Lambda_L = (-L, L)^d$ with Dirichlet boundary conditions.

A Wegner estimate is an upper bound of the expected number of eigenvalues in an interval $[a, b]$ of the form $C[a, b]^s L^{dm}$ with constants $C > 0$, $s \in (0, 1]$ and $m \geq 1$. It provides an important ingredient for a proof of localization, i.e. almost sure absence of continuous spectrum.

This is a joint work with Ivica Nakić, Matthias Täufer and Ivan Veselić.