Equidistribution, Uniform distribution: in between number theory and probability

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Abstract

The theory of equidistribution is about hundred years old, and has been developed primarily by number theorists and theoretical computer scientists. One purpose of this talk is to provide a brief introduction to equidistribution from a probabilist's viewpoint. We proceed by recalling a perspective on complete equidistribution, proposed by Nedžad Limić in 2002. Using it, various new examples of completely uniformly distributed mod 1 sequences, in the "metric" (meaning almost sure) sense, can be easily exhibited.

The translation from number theory to probability language brings into focus a version of the strong law of large numbers for weakly correlated complex-valued random variables, the study of which was initiated by Weyl in 1916, followed up by Davenport, Erdös and Leveque in 1963, dogded by Knuth in 1965, and greatly extended by Lyons in 1988. In this context, an applications to ∞-distributed Koksma's numbers $t^k \bmod 1$, $k \geq 1$ (where $t \in [1, a]$ for some $a > 1$), and an important generalization by Niederreiter and Tichy from 1985 will be discussed.

Interesting open questions remain. Based on an expository article, written in collaboration with N. Limić.