Complex Friedrichs systems and applications

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Abstract. Symmetric positive systems of first-order linear partial differential equations were introduced by Kurt Otto Friedrichs (1958) while treating equations that change their type, like the equations modelling transonic fluid flow. This system, today also known as a Friedrichs system, appeared to be convenient for the numerical treatment of various boundary value problems which also inspired Ern, Guermond and Caplain (2007) to express the theory in terms of operators acting in abstract Hilbert spaces obtaining well-posedness result in this abstract setting. Although some evolution (non-stationary) problems can be treated within this framework, their theory is not suitable for evolution problems like the initial-boundary value problem for the non-stationary Maxwell system, or the Cauchy problem for the symmetric hyperbolic system. Some numerical treatment of such non-stationary problems was done by Burman, Ern and Fernandez (2010), while the existence and uniqueness result was recently provided by Burazin and Erceg.

Most classical papers deal with Friedrichs systems in real space setting. We extend both stationary and non-stationary theory to the complex Hilbert space setting and, more general, to complex Banach spaces. We also prove the existence and uniqueness result taking the pivot space $H^s(\mathbf{R^d};\mathbf{C^r})$ instead of $L^2(\mathbf{R^d};\mathbf{C^r})$. Besides linear, semilinear problems can also be treated in complex case. We apply these results to some tipical examples of complex systems of partial differential equation, such as Dirac, Dirac-Klein-Gordon and Dirac-Maxwell system.

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