Defect distributions applied to differential equations with power function type coefficients

Ivana Vojnović

Department of Mathematics and Informatics
University of Novi Sad

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MiTPDE

- H-distributions (Antonić, Mitrović, 2011.) - $L^p - L^q$ spaces, $p = \frac{q}{q-1}$, $1 < p < \infty$, $u_n \rightharpoonup 0$ in $L^p(\mathbb{R}^d)$, $v_n \rightharpoonup 0$ in $L^q(\mathbb{R}^d)$, $\psi \in C^\kappa(\mathbb{S}^{d-1})$

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- H-distributions - $W^{-k,p} - W^{k,q}$, $H^p_{-s} - H^q_{-s}$ spaces, $s \in \mathbb{R}, 1 < p < \infty$ (Aleksić, Pilipović, V.)

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Theorem

If a sequence $u_n \rightharpoonup 0$ weakly in $W^{-k,p}(\mathbb{R}^d)$ and $v_n \rightharpoonup 0$ weakly in $W^{k,q}(\mathbb{R}^d)$, then there exist subsequences $(u_{n'}), (v_{n'})$ and a distribution μ such that for every $\varphi_1, \varphi_2 \in \mathcal{S}(\mathbb{R}^d)$, $\psi \in C^\kappa(\mathbb{S}^{d-1})$, $\kappa = [d/2] + 1$,

$$\lim_{n' \to \infty} \langle \varphi_1 u_{n'} , \overline{\mathcal{A}_{\overline{\psi}}(\varphi_2 v_{n'})} \rangle = \langle \mu, \varphi_1 \bar{\varphi}_2 \psi \rangle.$$

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- $\mathcal{S}(\mathbb{R}^d) \hat{\otimes} \mathcal{E}(\mathbb{S}^{d-1}) = \mathcal{SE}(\mathbb{R}^d \times \mathbb{S}^{d-1})$.

Unbounded symbols

- For weakly convergent sequences in $W^{-k,p} - W^{k,q}$ spaces multiplier (symbol) ψ is a bounded function, $\psi \in C(\mathbb{S}^{d-1})$ or $\psi \in C^{\kappa}(\mathbb{S}^{d-1})$

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- Function $a \in C^{\infty}(\mathbb{R}^d \times \mathbb{R}^d)$ is in $S^m_{1,0}$ class of symbols if for all $\alpha, \beta \in \mathbb{N}^d_0$,

$$|\partial^{\alpha}_{\xi} \partial^{\beta}_{x} a(x,\xi)| \leq c_{\alpha,\beta}(1 + |\xi|^2)^{\frac{m-|\alpha|}{2}}.$$

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  Notation: $\langle\xi\rangle = (1+|\xi|^2)^{1/2}$
- Let $m \in \mathbb{R}$, $N \in \mathbb{N}_0$. Then we consider the space $s^m_{\infty,N}$ of all $\psi \in C^N(\mathbb{R}^d)$ such that

$$|\psi|_{s^m_{\infty,N}} := \max_{|\alpha|\leq N} \|\partial^{\alpha}_{\xi}\psi(\xi)\langle\xi\rangle^{-m+|\alpha|}\|_{L^{\infty}} < \infty.$$

H-distributions with symbol $\psi \in s^m_{\infty,N}$

We fix $\psi \in s^m_{\infty,N}$, $N \geq 3d+5$. Then $\mathcal{A}_\psi : H^q_{m+s}(\mathbb{R}^d) \to H^q_s(\mathbb{R}^d)$ is continuous.

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$$\lim_{n\to\infty} \langle \varphi_1 u_n, \overline{\mathcal{A}_{\bar{\psi}}(\varphi_2 v_n)} \rangle = \langle \mu_\psi, \varphi_1 \bar{\varphi}_2 \rangle.$$

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Weight functions

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Definition (Morando, Nicola, Rodino)

Positive function $\Lambda \in C^{\infty}(\mathbb{R}^N)$ is a weight function if the following conditions are satisfied:

1. There exist positive constants $1 \leq \mu_0 \leq \mu_1$ and $c_0 < c_1$ such that
$$c_0 \langle z \rangle^{\mu_0} \leq \Lambda(z) \leq c_1 \langle z \rangle^{\mu_1}, \quad z \in \mathbb{R}^N;$$
2. There exists $\omega \geq \mu_1$ such that for any $\alpha \in \mathbb{N}_0^N$ and $\gamma \in \mathbb{K}_N \equiv \{0,1\}^N$
$$|z^{\gamma} \partial^{\alpha+\gamma} \Lambda(z)| \leq C_{\alpha,\gamma} \Lambda(z)^{1-\frac{1}{\omega}|\alpha|}, \quad z \in \mathbb{R}^N.$$

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2. Multi-quasi-elliptic polynomial:

$$\Lambda_{\mathcal{P}}(z) = \Big(\sum_{\alpha \in V(\mathcal{P})} z^{2\alpha} \Big)^{\frac{1}{2}},\ z \in \mathbb{R}^N.$$

Here $\mathcal{P}$ is a given complete polyhedron with the set of vertices $V(\mathcal{P})$.

Definition

Let $m \in \mathbb{R},\ \rho \in (0, 1/\omega]$. We denote by $M\Gamma^{m}_{\rho,\Lambda}$ the space of functions $a \in C^{\infty}(\mathbb{R}^{2d})$ such that for all $\alpha, \beta \in \mathbb{N}_0^d$, $\gamma_1, \gamma_2 \in \{0, 1\}^d$ it holds that

$$|x^{\gamma_1}\xi^{\gamma_2}\partial_{\xi}^{\alpha+\gamma_2}\partial_{x}^{\beta+\gamma_1}a(x,\xi)| \leq C_{\alpha,\beta,\gamma_1,\gamma_2}\Lambda(x,\xi)^{m-\rho|\alpha+\beta|}.$$

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$S^m_{1,0}$: $a \in C^\infty(\mathbb{R}^{2d})$ and for all $\alpha, \beta \in \mathbb{N}_0^d$

$$|\partial_\xi^\alpha \partial_x^\beta a(x,\xi)| \leq c_{\alpha,\beta}\langle\xi\rangle^{m-|\alpha|}.$$

We equip $M\Gamma^m_{\rho,\Lambda}$ with the family of norms

$$\|a\|_{M\Gamma^m_k} = \sup_{|\alpha|+|\beta|\le k, \gamma\in\mathbb{K}} \sup_{(x,\xi)\in\mathbb{R}^{2d}} \frac{|x^{\gamma_1}\xi^{\gamma_2}\partial_\xi^{\alpha+\gamma_2}\partial_x^{\beta+\gamma_1} a(x,\xi)|}{\Lambda(x,\xi)^{m-\rho|\alpha+\beta|}},$$

where $k\in\mathbb{N}_0$, $\gamma=(\gamma_1,\gamma_2)$, $\gamma_i\in\mathbb{K}_d, \alpha,\beta\in\mathbb{N}_0^d$.
Pseudo-differential operator T_a with a symbol $a\in M\Gamma^m_{\rho,\Lambda}$ is defined by

$$T_a u(x) := \int_{\mathbb{R}^d} e^{ix\cdot\xi} a(x,\xi)\hat{u}(\xi) đ\xi, \quad u\in\mathcal{S}(\mathbb{R}^d).$$

Let $\Lambda(x,\xi)$ be a weight function, $s \in \mathbb{R}$, $1 < p < \infty$. We denote by $H_{\Lambda}^{s,p}(\mathbb{R}^d)$ the space of all $u \in \mathcal{S}'(\mathbb{R}^d)$ such that $T_{\Lambda^s} u \in L^p(\mathbb{R}^d)$.
Since $\Lambda(x,\xi)^s$ is elliptic of order s there exists an operator $T_b \in ML_{\rho,\Lambda}^{-s}$ such that

$$T_b T_{\Lambda^s} = I + R_s,$$

where R_s is a regularizing operator. We define norm on $H_{\Lambda}^{s,p}$ in the following manner:

$$\|u\|_{s,p,\Lambda} = \|T_{\Lambda^s} u\|_{L^p} + \|R_s u\|_{L^p}.$$

With this norm $H_{\Lambda}^{s,p}(\mathbb{R}^d)$ becomes a Banach space.

Theorem

If $b \in M\Gamma^m_{1/\omega,\Lambda}$, *then* $T_b : H^{s+m,p}_\Lambda(\mathbb{R}^d) \to H^{s,p}_\Lambda(\mathbb{R}^d)$ *continuously for* $s, m \in \mathbb{R}$ *and* $1 < p < \infty$. *We have the following estimate*

$$\|T_b u\|_{H^{s,p}_\Lambda} \leq C \|b\|_{M\Gamma^m_k} \|u\|_{H^{s+m,p}_\Lambda},$$

for some $k \in \mathbb{N}, k > 2d$.

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Theorem (Lizorkin-Marcinkiewicz)

Let $m(\xi)$ *be continuous together with derivatives* $\partial^\gamma_\xi m(\xi)$*, for any* $\gamma \in \{0,1\}^d$*. If there is a constant* $c > 0$ *such that*

$$\xi^\gamma \partial^\gamma_\xi m(\xi) \leq c, \ \xi \in \mathbb{R}^d, \ \gamma \in \{0,1\}^d,$$

then for $1 < p < \infty$ *there exists a constant* $B = B(p,d)$ *such that* $\|T_m u\|_{L^p} \leq B\|u\|_{L^p}, \ \ u \in \mathcal{S}(\mathbb{R}^d)$.

To obtain L^p-boundedness it is enough to assume that for $a(x, \xi)$ it holds that

$$|\xi^\gamma \partial_x^\lambda \partial_\xi^{\nu+\gamma} a(x, \xi)| \leq C \langle \xi \rangle^{-\varepsilon|\nu|}, \quad (x, \xi) \in \mathbb{R}^{2d},$$

for some $\varepsilon > 0$, and for all $\lambda, \nu \in \mathbb{N}_0^d$, $\gamma \in \mathbb{K}_d$.

Theorem

Let $v \in H_\Lambda^{m,q}(\mathbb{R}^d)$, $m \in \mathbb{R}, 1 < q < \infty$ and $\varphi \in \mathcal{S}(\mathbb{R}^d)$. Then $\varphi v \in H_\Lambda^{m,q}(\mathbb{R}^d)$.

We denote by $(M\Gamma^m_{\rho,\Lambda})_0 \subset M\Gamma^m_{\rho,\Lambda}$ the space of symbols $\psi \in M\Gamma^m_{\rho,\Lambda}$ such that for all $(\alpha_1, \alpha_2) \in \mathbb{N}_0^{2d},\ (\gamma_1, \gamma_2) \in \mathbb{K}_{2d}$ (resp. $\gamma \in \mathbb{K}_d$)

$$\lim_{n\to\infty} \sup_{|(x,\xi)|\geq n} \frac{|x^{\gamma_1}\xi^{\gamma_2}\partial^{(\alpha_1,\alpha_2)+(\gamma_1,\gamma_2)}\psi((x,\xi))|}{\Lambda(x,\xi)^{m-\rho(|\alpha_1|+|\alpha_2|)}} = 0.$$

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Theorem

Let $u_n \rightharpoonup 0$ in $L^p(\mathbb{R}^d)$ and $v_n \rightharpoonup 0$ in $H^{m,q}_\Lambda(\mathbb{R}^d)$, $m \in \mathbb{R}$, $\rho = 1/\omega$. Then, up to a subsequence, there exists a distribution $\mu \in (\mathcal{S}(\mathbb{R}^d)\hat{\otimes}(M\Gamma^m_{\rho,\Lambda})_0)'$ (resp., $\mu \in (\mathcal{S}(\mathbb{R}^d)\hat{\hat{\otimes}}(\widetilde{M\Gamma^m_{\rho,\Lambda}})_0)'$) such that for all $\varphi \in \mathcal{S}(\mathbb{R}^d)$ and all $\psi \in (M\Gamma^m_{\rho,\Lambda})_0$ (resp., $\psi \in (\widetilde{M\Gamma^m_{\rho,\Lambda}})_0$),

$$\lim_{n\to\infty}\langle u_n, \overline{T_{\bar\psi}(\varphi v_n)}\rangle = \langle \mu, \bar\varphi \otimes \psi\rangle.$$

Theorem

Let $u_n \rightharpoonup 0$ *in* $L^p(\mathbb{R}^d)$ *and* $v_n \rightharpoonup 0$ *in* $H^{m,q}_{\Lambda}(\mathbb{R}^d)$, $m \in \mathbb{R}$. *Assume that* $\psi \in M\Gamma^m_{1/\omega,\Lambda}$. *Then, up to subsequences, there exists a distribution* $\mu_\psi \in \mathcal{S}'(\mathbb{R}^d)$ *such that for all* $\varphi \in \mathcal{S}(\mathbb{R}^d)$,

$$\lim_{n\to\infty} \langle u_n, \overline{T_{\bar{\psi}}(\varphi v_n)} \rangle = \langle \mu_\psi, \bar{\varphi} \rangle.$$

Theorem

Let $u_n \rightharpoonup 0$ *in* $L^p(\mathbb{R}^d)$*. Assume that*

$$\lim_{n\to\infty} \langle u_n, T_{\Lambda(x,\xi)^m}(\varphi v_n)\rangle = 0,$$

for every sequence $v_n \rightharpoonup 0$ *in* $H_\Lambda^{m,q}(\mathbb{R}^d)$*,* $m \in \mathbb{R}$*. Then for every* $\theta \in \mathcal{S}(\mathbb{R}^d)$*,* $\theta u_n \to 0$ *strongly in* $L^p(\mathbb{R}^d)$*.*

Theorem

Let $u_n \rightharpoonup 0$ in $L^p(\mathbb{R}^d)$. Assume that

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for every sequence $v_n \rightharpoonup 0$ in $H_\Lambda^{m,q}(\mathbb{R}^d)$, $m \in \mathbb{R}$. Then for every $\theta \in \mathcal{S}(\mathbb{R}^d)$, $\theta u_n \to 0$ strongly in $L^p(\mathbb{R}^d)$.

Corollary

Let $u_n \rightharpoonup 0$ in $L^p(\mathbb{R}^d)$ and $a \in EM\Gamma^m_{\rho,\Lambda}$. Assume that

$$\lim_{n\to\infty} \langle u_n, T_a(\varphi v_n)\rangle = 0,$$

for every sequence $v_n \rightharpoonup 0$ in $H_\Lambda^{m,q}(\mathbb{R}^d)$, $m \in \mathbb{R}$. Then for every $\theta \in \mathcal{S}(\mathbb{R}^d)$, $\theta u_n \to 0$ strongly in $L^p(\mathbb{R}^d)$.

Let

$$P(x,D)u_n = \sum_{(\alpha,\beta)\in V(\mathcal{P})} x^\beta D_x^\alpha u_n = f_n, \qquad (1)$$

for some complete polyhedron $\mathcal{P}$, where $u_n \rightharpoonup 0$ in $H^{1,p}_{\mathcal{P}}$ and $\varphi f_n \to 0$ in $L^p(\mathbb{R}^d)$ for every $\varphi \in \mathcal{S}(\mathbb{R}^d)$. Here $V(\mathcal{P})$ denotes the set of vertices of $\mathcal{P}$ and $p(x,\xi) = \sum_{(\alpha,\beta)\in V(\mathcal{P})} x^\beta \xi^\alpha \in M\Gamma^1_{1/\omega,\mathcal{P}}$.

Theorem

Let $u_n \rightharpoonup 0$ in $H^{1,p}_{\mathcal{P}}(\mathbb{R}^d)$ satisfies (1). Then for any $v_n \rightharpoonup 0$ in $L^q(\mathbb{R}^d)$ it holds that

$$\mu_p = 0 \quad \text{in } \mathcal{S}'(\mathbb{R}^d).$$

If p is elliptic, then $\theta u_n \to 0$ in $H^{1,p}_{\mathcal{P}}$, for every $\theta \in \mathcal{S}(\mathbb{R}^d)$.

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