We will show that a Fourier multiplier with a symbol $a \in S^0$ is bounded.

Example.

Let $L = [-\pi, \pi]$. We have

$$\int_{-\pi}^{\pi} |\hat{f}(\xi)|^2 \, d\xi = \int_{-\pi}^{\pi} \left| \sum_{n \in \mathbb{Z}} a(n) e^{in\xi} \right|^2 \, d\xi$$

in some applications, when a flow occurs in the highly heterogeneous porous media (e.g. in the GOs (14) equation problems [?]), we can shift coefficients and then in the resulting model.

Preliminaries from matrix analysis

Clearly, it is a regular change of variables and it holds

$$\eta = (\eta_1, \eta_2, \ldots, \eta_k)$$

in inverse is given by

$$\lambda = \left( \lambda_1 + \eta_1, \lambda_2 + \eta_2, \ldots, \lambda_k + \eta_k \right).$$

Since $A$ is only assumed to be non-negative definite, we can not obtain the bound of $|\lambda|^2$ only terms of $A$. For matrix $M$ one easily gets $\|M\| \leq \max \{|A_{ij}|\}$ and $|\lambda|$. The case where $A$ depends continuously only one parameter, we get that the corresponding norms depend continuously on $a$ as well.

Preliminaries from matrix analysis II

Fourier multipliers I

Let $a \in \mathbb{R}^3$ be a non-negative definite matrix. Let

$$\gamma = (\gamma_1, \gamma_2, \gamma_3) \in R^3$$

By [11], a problem of type (11) was considered, but with that and diffusion independent of $x$ and $u$. Hence the geometry of the flux allows the separation of coefficients from the unknown $u$, by applying the Fourier transform. In our work (in progress) we consider the arbitrary, non-linear terms.

In [1], a problem of type (11) was considered, but with that and diffusion independent of $x$ and $u$. Hence the geometry of the flux allows the separation of coefficients from the unknown $u$, by applying the Fourier transform. In our work (in progress) we consider the arbitrary, non-linear terms.

H-measures

Theroem. If $\{u_n\} \subset L^2(\Omega \times [0, T])$ is a bounded sequence of solutions to the Cauchy problem (12) with $\mathbb{C} \subset \mathbb{R}$, then $u_n \rightharpoonup u$ in $L^2(\Omega \times [0, T])$ and $u$ is a solution of (12) in the sense of the limit passage.

H-measures are not only derivatives of the same highest order. For example, we can change the scaling and put $\chi \frac{\partial}{\partial t} (u_{\frac{\cdot}{\cdot}}) \frac{\partial}{\partial x} (u_{\frac{\cdot}{\cdot}})$ instead of $\chi \frac{\partial}{\partial t} u$, but such H-measures will be able to see the first order derivatives with respect to $(x_1, \ldots, x_n)$ and second order derivatives with respect to $(x_1, \ldots, x_n)$.

In other words, no change of the highest order of the equation is permitted. We overcome this situation by considering multiple multiplier operators with symbols of the form

$$\left( \sum_{\alpha \in \mathbb{N}_0^n} a_{\alpha}(x) \frac{\partial^\alpha}{\partial x^\alpha} \right) u$$

where the matrix $A$ represents the diffusion matrix in the degenerate parabolic equation.

Marzickiewicz multiplier theorem

Corollary. Suppose that $A \in \mathbb{C}^{n \times n}$ is a bounded matrix such that for some constant $C > 0$ it holds

$$\|A\| \leq C.$$ 

For every multi-index $\alpha = (\alpha_1, \ldots, \alpha_n) \in \mathbb{N}_0^n$, such that $|\alpha| = \alpha_1 + \ldots + \alpha_n$, we assume that the function $u$ is an $L^2$-multiplier for $|\xi|^2$, and the operator norm of $A$ is bounded. Specifically, $A \in C^{\infty}$, where $C^{\infty}$ denotes only on $A$ and $\alpha$.

Lemma. $B$ is a symbol of a multiplier bounded in $L^2(\mathbb{R}^n)$, then the definition is extended by $\chi(x, \xi) u \in L^2(\mathbb{R}^n)$, where $(\xi, \eta) \in \mathbb{R}^n$. The definition of multipliers bounded on $L^2$ with the same operator norm as $A$.

H-measures II

Corollary. Let $u \in L^2(\Omega \times [0, T])$ be the function defined in the previous Corollary. Let $\tilde{E} \subset \Omega \times [0, T]$ be a radial compact support of the function $F$. If $\tilde{E} \subset \Omega \times [0, T]$ is such that for some $r \in (0, T)$ we have $r^2 \leq \tilde{E}$, then

$$\|u\|_{L^2(\Omega \times [0, T])} \leq C.$$

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